Example Probability Calculations (Standard Normal)

1.) Compute $P(Z < 1.5)$

$p = \text{pnorm}(1.5)$

2.) Compute $P(Z > 1.5)$

$p = P(Z > 1.5) = 1 - P(Z < 1.5)$

$= 1 - \text{pnorm}(1.5)$

3.) Compute $P(-1 < Z < 1.5)$

$p = P(-1 < Z < 1.5) = P(Z < 1.5) - P(Z < -1)$

$= \text{pnorm}(1.5) - \text{pnorm}(-1)$

4.) Compute $P(Z < -1 \text{ or } Z > 1.5)$

$p = P(Z < -1 \text{ or } Z > 1.5) = P(Z < -1) + P(Z > 1.5)$

$= \text{pnorm}(-1) + [1 - \text{pnorm}(1.5)]$

5.) Compute $P(|Z| < 1.5)$

$p = P(|Z| < 1.5) = P(-1.5 < Z < 1.5)$

$= \text{pnorm}(1.5) - \text{pnorm}(-1.5)$

5b) Compute $P(|Z| < 1.5)$ using symmetry

Two red areas have equal prob.

$P(-1.5 < Z < 0) = P(0 < Z < 1.5)$

Two green areas have equal prob.

$P(Z < -1.5) = P(1.5 < Z)$

$P(-1.5 < Z < 0) = 1/2 - P(Z < -1.5)$

$p = P(|Z| < 1.5) = 2\left[P(-1.5 < Z < 0)\right]$

$= 2\left[1/2 - P(Z < -1.5)\right]$

$= 2\left[1/2 - \text{pnorm}(-1.5)\right]$ Ans #1

$P(0 < Z < 1.5) = P(Z < 1.5) - 1/2$

$p = P(|Z| < 1.5) = 2\left[P(0 < Z < 1.5)\right]$

$= 2\left[P(Z < 1.5) - 1/2\right]$

$= 2\left[\text{pnorm}(1.5) - 1/2\right]$ Ans #2

6) Compute $P(|Z| > 1.5)$ using symmetry

$p = P(|Z| > 1.5) = P(Z < -1.5) + P(Z > 1.5)$

$= 2\left[P(Z < -1.5)\right]$

$= 2\left[\text{pnorm}(-1.5)\right]$

7.) Find critical z-value so that $P(Z < z) = .07$

$z = \text{qnorm}(.07)$

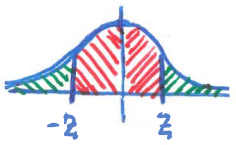
8.) Find critical z-value so that $P(Z > z) = .07$

$P(Z < z) = 1 - P(Z > z)$

$= 1 - .07$

$z = \text{qnorm}(1 - .07)$ ← "qnorm(.93)"

9) Find critical z-value so that $P(|Z| > z) = .07$



Two red areas have equal prob.

$P(-z < Z < 0) = P(0 < Z < z)$

Two green areas have equal prob.

$P(Z < -z) = P(z < Z)$

→ If $P(|Z| > z) = .07$, then what is $P(Z < z)$?

$P(Z > z) = \frac{1}{2}[P(|Z| > z)]$

$P(Z < z) = 1 - P(Z > z)$

$= 1 - \frac{1}{2}[P(|Z| > z)]$

$= 1 - \frac{1}{2}[.07]$

$z = \text{qnorm}(1 - \frac{1}{2}[.07])$

10) Find the critical z-value so that

$P(|Z| < z) = .07$

→ If $P(|Z| < z) = .07$, then what is $P(Z < z)$?

$P(0 < Z < z) = \frac{1}{2} P(|Z| < z)$

$P(Z < z) = \frac{1}{2} + P(0 < Z < z)$

$= \frac{1}{2} + [\frac{1}{2} P(|Z| < z)]$

$= \frac{1}{2} + \frac{1}{2}(.07)$

$z = \text{qnorm}(\frac{1}{2} + \frac{1}{2}(.07))$ ← Ans #1

$P(|Z| > z) = 1 - P(|Z| < z)$

$= 1 - .07 = .93$

$P(Z < z) = 1 - P(Z > z)$

$= 1 - \frac{1}{2} P(|Z| > z)$

$= 1 - \frac{1}{2}[.93]$

$z = \text{qnorm}(1 - \frac{1}{2}(.93))$ ← Ans #2

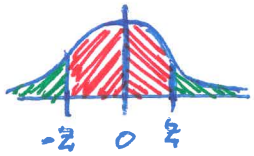
$1 - \frac{1}{2}[1 - .07] = 1 - \frac{1}{2} + \frac{1}{2}(.07)$

$= \frac{1}{2} + \frac{1}{2}(.07)$

Same as Ans #1

9b) Find z so that $P(|Z| > z) = .07$ (use $P(Z < -z)$)

→ If $P(|Z| > z) = .07$, then what is $P(Z < -z)$?



$$P(|Z| > z) = P(Z < -z) + P(Z > z)$$
$$= 2P(Z < -z)$$
$$P(Z < -z) = \tfrac{1}{2} P(|Z| > z)$$
$$= \tfrac{1}{2}(.07)$$
$$z = -\text{gnorm}(\tfrac{1}{2}(.07))$$

10b) Find z so that $P(|Z| < z) = .07$ (use $P(Z < -z)$)

$$P(|Z| > z) = 1 - P(|Z| < z)$$
$$= 1 - .07 = .93$$
$$P(Z < -z) = \tfrac{1}{2} P(|Z| > z)$$
$$= \tfrac{1}{2}(.93)$$
$$z = -\text{gnorm}(\underline{\tfrac{1}{2}(.93)})$$

$$\tfrac{1}{2}(1 - .07) = \tfrac{1}{2} - \tfrac{1}{2}(.07)$$
$$= \tfrac{1}{2} - P(-z < Z < 0)$$

Remark: When looking for critical z-values of the form $P(|Z| > z)$ or $P(|Z| < z)$ it is often much easier to find $-z$ first

- $P(Z < -z) = \tfrac{1}{2} P(|Z| > z)$
- $P(Z < -z) = \tfrac{1}{2}[1 - P(|Z| < z)]$